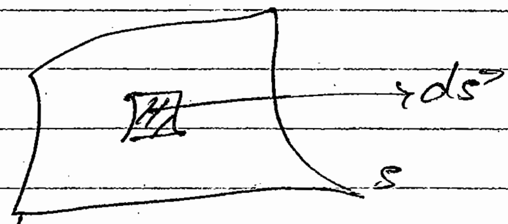
$\iint_S \vec{A}\cdot d\vec{s}$ = Flux of the vector field A through the surface s.

* **Surface Integration :-** It is a integration of a vector on a open or a close surface.

Any surface that incloses some volume is close surface, e.g. sphere, cube etc.

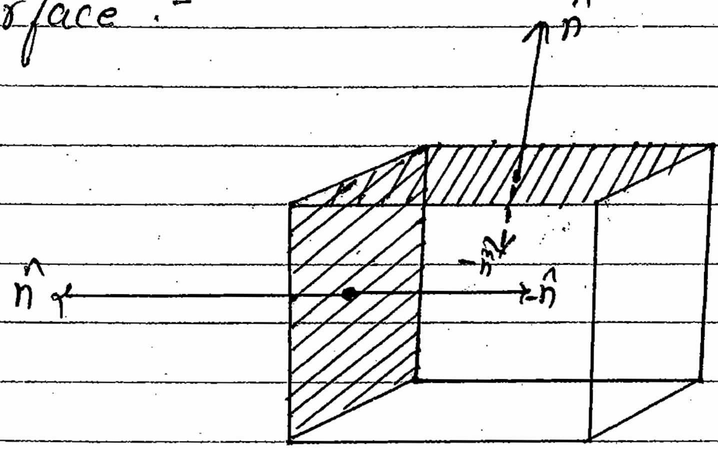
$$\iint_S \vec{A}\cdot d\vec{s} = \iint_S \vec{A}\cdot\hat{n}\, ds$$



$\hat{n}$ is the direction of $\vec{A}$. And $d\vec{s}$ is magnitude of element.

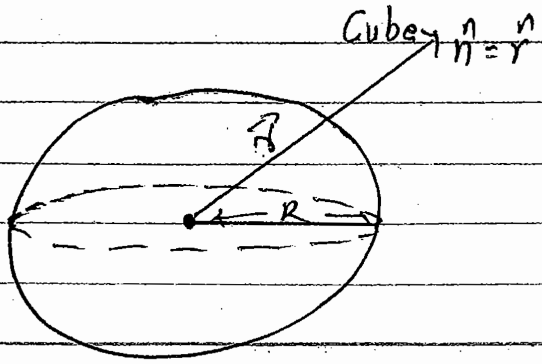
$\hat{n}$ is a unit vector normal to the surface s indicating the +ve direcⁿ of surface.

1. Close Surface :-

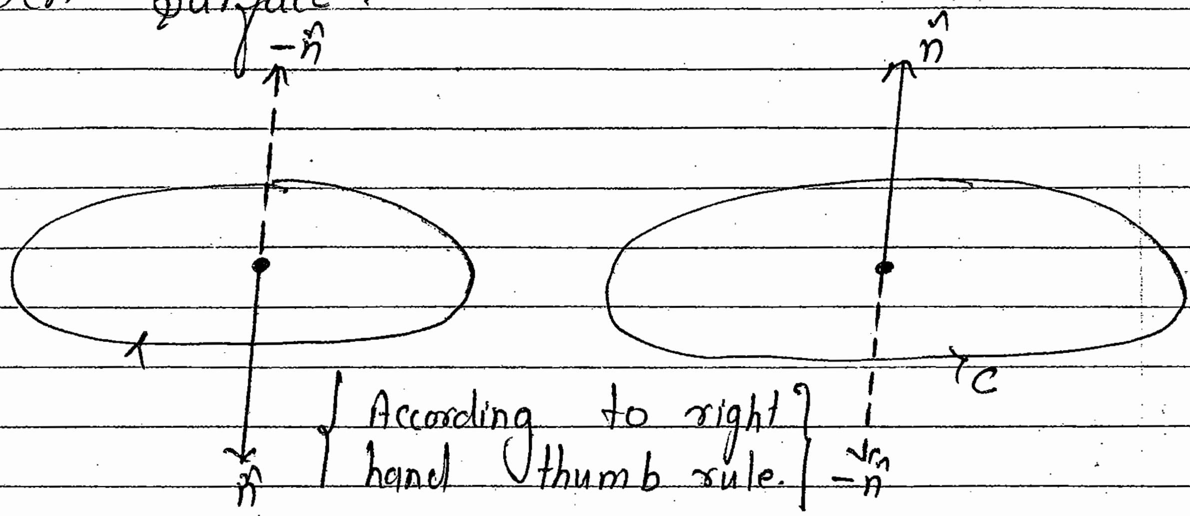


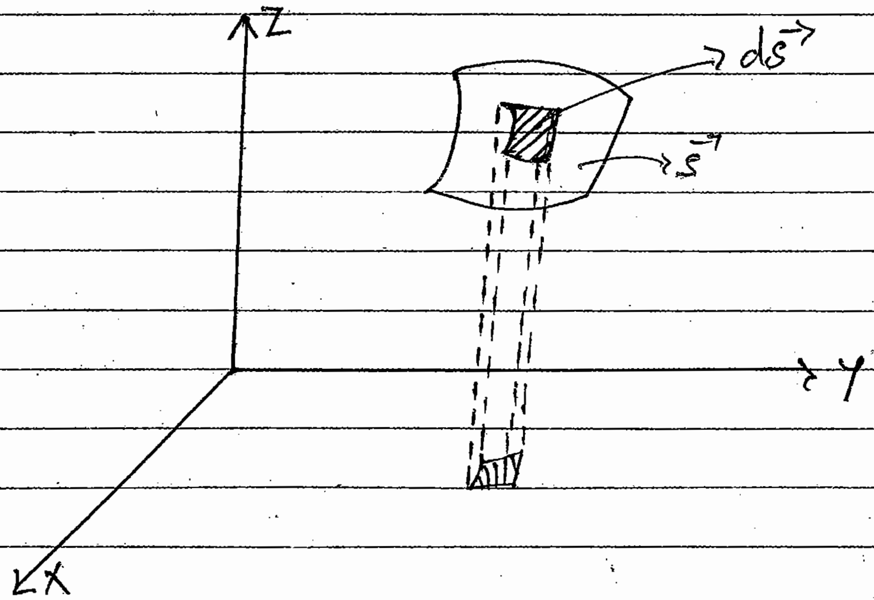
Cube

2. Sphere :-



2. Open Surface :-





If the surface of integration is not parallel to any one of the co-ordinate plane then we have to draw the projection on any one of co-ordinate plane.

~~Eta~~ Projection of $\vec{ds}$ in xy-plane = dxdy

$\Rightarrow \quad ds.\hat{k} = dxdy$ $\quad$ ∵ $\vec{A}$ $\hat{B}$ — $\vec{ds}$ xy-plane's dir^n

$\Rightarrow \quad \hat{n}\, ds.\hat{k} = dxdy$

$\Rightarrow \quad ds\,(\hat{n}.\hat{k}) = dxdy$

$$\Rightarrow \boxed{ds = \frac{dx\,dy}{\hat{n}\cdot\hat{k}}}$$

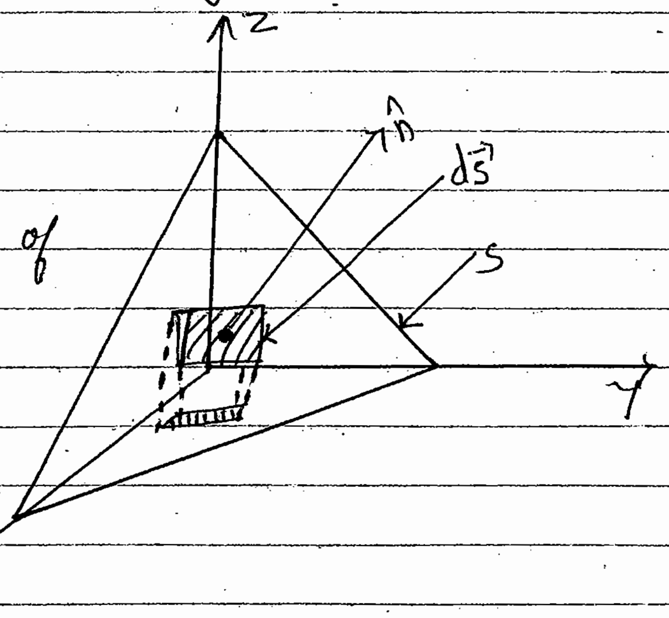
$$\boxed{\iint_S \vec{A}\cdot\hat{n}\,d\vec{s} = \iint \vec{A}\cdot\hat{n}\,\frac{dx\,dy}{|\hat{n}\cdot\hat{k}|}}$$

Here $|\,|$ mod is becoz $\hat{n}\cdot\hat{k}$ is area and area is always +ve.

Ex $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ Calculate $\iint \vec{A}\cdot\hat{n}\,d\vec{s}$
S is surface of plane $2x+3y+6z=12$ in the first octant.

Sol^n $\frac{x}{6} + \frac{y}{4} + \frac{z}{2} = 1$

∵ We know in 3-D eq^n of the plane is -
$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$



$$\iint \vec{A}\cdot\hat{n}\,ds = \iint \vec{A}\cdot\hat{n}\,\frac{dx\,dy}{|\hat{n}\cdot\hat{k}|}$$

$S : 2x + 3y + 6z = 12$

$$\hat{n} = \frac{\vec{\nabla}S}{|\vec{\nabla}S|} = \frac{2\hat{i}+3\hat{j}+6\hat{k}}{7}$$

$$\hat{n}\cdot\hat{k} = \frac{6}{7}$$

$$\vec{A}\cdot\hat{n} = \frac{36z - 36 + 18y}{7}$$

$$\therefore \quad z = \frac{12 - 2x - 3y}{6}$$